Computer Model Clarifies Spontaneous Charge Redistribution in Conductors

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COMPUTER MODEL CLARIFIES SPONTANEOUS CHARGE REDISTRIBUTION IN CONDUCTORS

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Since powerful microcomputers with good graphics capabilities became widely available in the last decade or so, their usefulness in explaining difficult physical concepts in pictures has motivated the development of a great deal of educational software. The subject of electromagnetism, for example, is one that most students at all levels find difficult, and much excellent software is available to help with its exposition.[1] We describe here a computation that should be a valuable addition to this list and also allow exploration of basic physics not easily addressed by traditional approaches.

In introductory electrostatics courses, an important area of study is the distribution of excess charge in material objects. For insulators, it is fairly easy to demonstrate and to explain that the excess charge is immobile and maintains its initial distribution until neutralized. For conductors, in which charge is mobile, there occurs a redistribution, the outcome of which is not intuitively obvious. Virtually all textbooks treat the problem in a similar fashion.[2, 3, 4] They model conductors as containing an unlimited reservoir of free (dissociable) charge carriers and develop the following structured sequence of propositions:

1. In electrostatics, the electric field is zero at all points inside a conductor (because of the reservoir of dissociable charges).
2. All points inside the conductor and on the surface are at the same potential (because the internal field is zero).
3. The electric field at the surface of the conductor is everywhere normal to the surface (a property of equipotential surfaces).
4. All excess charge resides on the surface of the conductor (from Gauss's law).
5. The surface charge density is greatest at sharp points. [Establishing this effect is quite difficult. The conventional proof involves equipotential surfaces of idealized geometries (see "Discussion" p. 44).]
6. The electric field is strongest just outside sharp points (again from Gauss's law).
7. The electric field is zero on the interior of a closed hollow conductor of any shape (from a thought experiment, removing material from the interior).

This sequence of deductions creates a number of difficulties. First, from the student's point of view, the propositions are not easily distinguished from the (relatively unfamiliar) concepts needed to deduce them. Few introductory students are comfortable in applying Gauss's law and equipotentials. Second, the dissociable charges in the conductor play a confusing role. They are critical for the first proposition but are shown to be irrelevant in the end. The whole argument has a "take away the number you first thought of" feel to it. Third, the congregation of excess charge at sharp points is quite counter-intuitive, and the standard explanations[5, 6, 7] are likely to be unconvincing and only add to the confusion.

In this paper, we outline a simple computer simulation of the charge redistribution process that demonstrates all the behavior outlined above and that uses only Coulomb's law.

Computational approach

To avoid the confusion introduced by the presence of dissociable charges, we use the following simplified model. Identical point charges are imagined to be placed inside a region of space bounded by a closed surface. These charges are free to move anywhere within that region but cannot cross the surface boundary and move outside it.

Qualitatively, much of the behavior of the system can easily be predicted. The charges will move apart from one another, and they continue to move outward until they hit the boundary. Once there, they can only move *along* the surface. All motion will cease when the net force on every particle has no component parallel to the surface. It is a small logical step from there to deduce that, if the number of charges is large enough, the total electric field at all points just outside the surface will be directed perpendicularly outward. From that deduction, it follows that the surface will be an equipotential.

What cannot be so easily predicted are answers to the following two questions:

- What is the electric field at points inside the volume?
- How are the charges distributed along the surface?

To answer these questions, a quantitative analysis must be performed.

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Clearly, for any plausible number of charges, this calculation must be done on a computer. The computation is quite straightforward in principle and could easily be done by students, as the only physical law is the Coulombic repulsion between pairs of like charges. In practice, however, the writing of such a program is complicated by the need for a graphical interface that allows the important results to be appreciated visually. In the authors' institution, students routinely program in Pascal, using the M.U.P.P.E.T. utilities,[8] and the diagrams below are generated by a program that makes extensive use of the graphical features of these utilities. However, it should be stressed that the actual results of the calculation do not depend on the particular presentation.

In writing such a program, the following points need to be borne in mind:

1. The boundary of the volume is conveniently modeled as a closed surface of arbitrary shape, which can be defined either by calculation or by manually drawing it on the screen with a mouse (see Fig. 1). It is numerically represented by a set of point coordinates (x_i, y_i) spaced an appropriately small distance apart, with the last and first point coordinates being identical $(x_N, y_N) = (x_1, y_1)$. Typically, such a surface is represented by about 100 points. For calculation purposes, the surface is considered to be continuous and interpolated between the points stored in the data structure representing the surface.

2. The calculation starts with a finite number of point charges located at random positions on the interior of the conductor (see Fig. 2). These might be placed individually with the mouse or placed automatically by calculation.

Coulomb's law is used to calculate the force on each charge. Because the surface is represented by a closed curve on a two-dimensional monitor screen, the most convenient interpretation is that both the surface and the charges extend indefinitely in the third dimension. Thus, the field produced by each charge has an effective $1/r$ dependence rather than inverse-square. It would be possible to compute a full three-dimensional simulation and display a cross section onscreen, but the advantage for the purposes of the demonstration would be minimal, and the computation would be more complex and therefore slower.

3. When the computation is performed, each charge is displaced in the direction of its force vector. It is not physically unrealistic to make the magnitude of the displacement proportional to the force $(d \propto F)$; in effect, this relationship assumes that the charges move as in a resistive medium, with drift velocity proportional to field strength. However, the result does not depend on the particular transport mechanism. Electrostatically, what matters is where the charges end up, not how they got there.

4. If the calculated value of a charge's displacement takes it outside the surface, that displacement must be truncated, and the charge is assumed to be on the surface. Further displacement is governed only by the tangential component of force. A suitable test for deciding when any charge has crossed the surface is to compare the direction of the difference vector between the charge and the nearest point on the surface with the outward normal to the surface at that point.

5. When it comes to visualizing the physical state of the system, the direction of the force on each charge may be

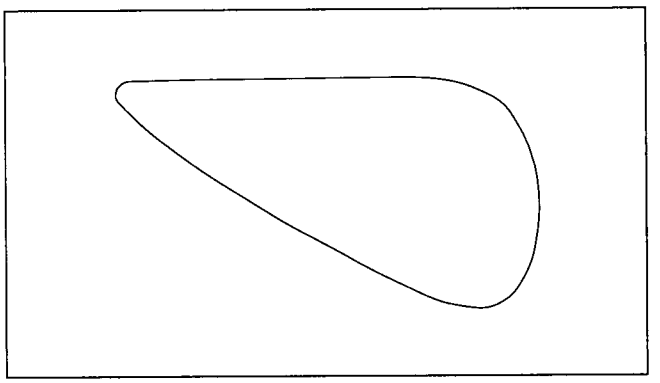

Figure 1. Arbitrarily shaped boundary represents a typical surface.

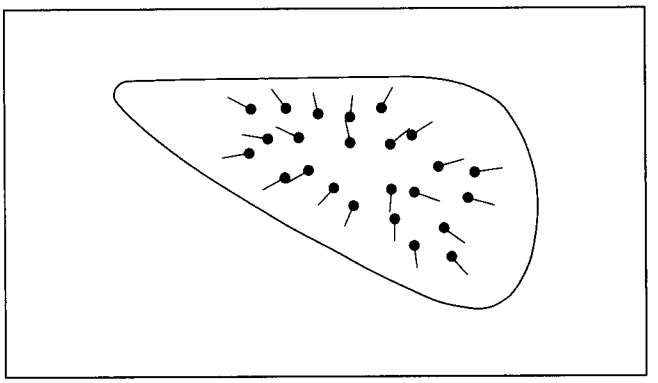

Figure 2. User positions charges randomly within the surface at the start of the computation. Short lines on the points describe the computed directions of the electrostatic forces.

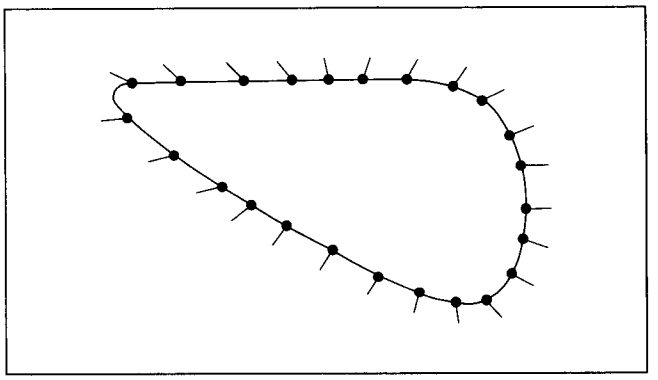

Figure 3. Charges move rapidly out to the surface.

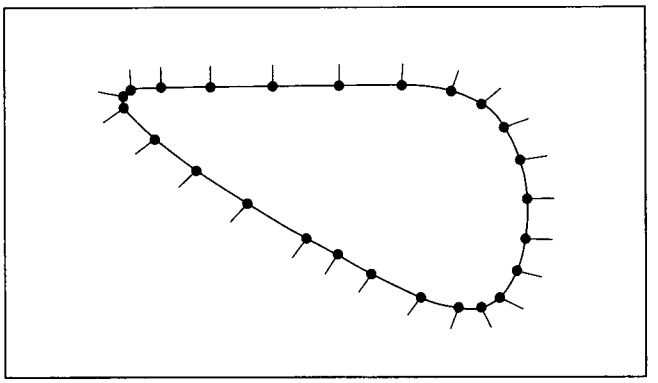

Figure 4. Charges migrate along the surface until the force vectors are perpendicular to it.

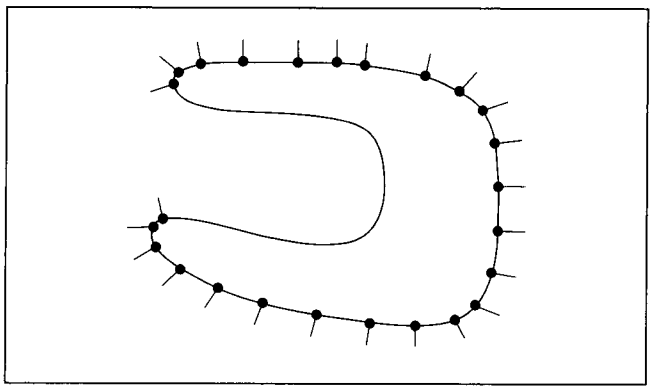

Figure 5. Program lets students observe that there is no charge at surface

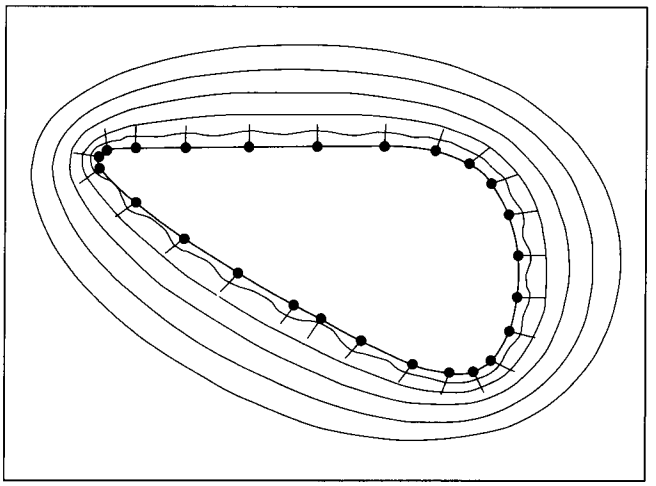

Figure 6. Computed equipotentials demonstrate that those near the physical surface have the same shape as and are approximately parallel to it.

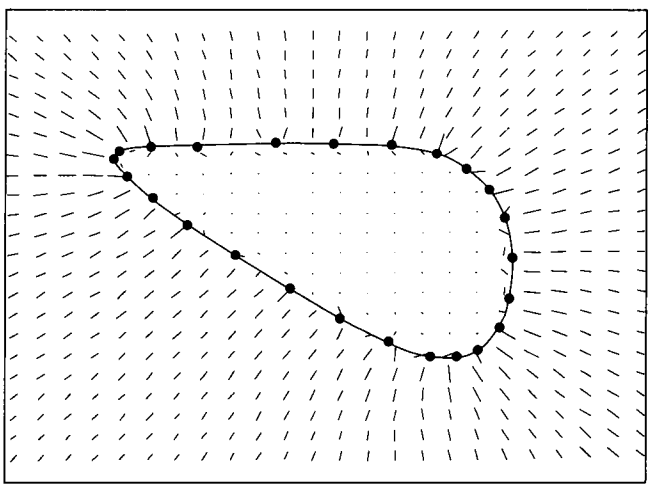

Figure 7. Electric field map demonstrates approximate cancellations of the field at interior points and its intensification just outside regions of high curvature, where the charges have congregated.

represented by a short line (as in Fig. 2). At a later stage, it might be useful to represent the magnitude of the electric field by the length of an arrow, and it would also prove worthwhile to write a procedure to draw lines of equipotential (as in the later Figs. 6 and 7).

Results

When a typical computation is done, the following results are observed. As expected, the charges quickly spread out and move to the surface with none remaining in the interior (see Fig. 3).

They continue to migrate along the surface until the tangential component of the net force for every charge is zero—that is, each force vector will be locally perpendicular to the surface, as expected (see Fig. 4).

Because any one of these mobile charges may be considered a "small test charge," the direction of its force arrow is aligned with the electric field at that point. Therefore, the following two basic propositions are immediately established:

- All excess charge resides on the surface.
- The electric field at the surface is everywhere perpendicular to it.

Just by inspection, the third conclusion can also be drawn, as follows:

- There is a higher concentration of charge near regions of smaller convex radius.

In fact, much more information can be gained than is usually available from the textbook approach. For example, at regions of concave curvature, the charge density is zero (see Fig. 5).

The next proposition, also expected, is that

- The surface is an equipotential.

A simple calculation of the potential at any point on the screen is not difficult, and a plot of equipotential surfaces can be drawn (see Fig. 6). The result conclusively demonstrates that the near-lying equipotentials are indeed closely parallel to the surface and become more and more so as the number of mobile charges is allowed to increase.

It is also straightforward to calculate the electric field strength and produce a map at a grid of points throughout the entire region (see Fig. 7). Two features are apparent from such a map. First, the external field strength is greatest immediately outside the surface regions of small convex radius. This feature is only to be expected, because the surface charge density is greatest in such regions, as also is the gradient of the potential. Second, and somewhat surprisingly, the electric field at all points on the interior of the volume is very small. The cancellation of the interior field becomes more and more pronounced as the number of mobile charges increases.

Discussion

The results we have reported here are not new. What is new is the way in which they are demonstrated. The approach is intrinsically easy for students to understand, because it assumes only Coulomb's law. Furthermore, the simulation demonstrates features of charge redistribution that are quite counterintuitive and otherwise only accessible by subtle argument. It also raises insightful questions about fundamental physics.

One such question is why charges tend to collect at sharp points. This phenomenon is easy to demonstrate experimentally but is difficult to explain. Many textbooks follow the discussion used by Feynman,[5] which shows the charge density to be greater on the smaller of two spheres joined by a long wire. Other derivations[6,7] use less-artificial idealizations of a conductor but much more opaque mathematics. All these approaches rely on the equipotential property of the conduct-

ing surface to explain charge accumulation. Unfortunately, in introductory physics courses, students are seldom sufficiently comfortable with the concept of potential to regard such explanations as credible in the face of the counterintuitive nature of the result.

The computational approach, on the other hand, relies only on simple physical principles—the Coulomb repulsion between mobile charge carriers. Note especially that the equipotential nature of the surface is not a precondition of the computation but one of its outcomes. Furthermore, the charge density can be quantitatively determined for any surface and not just for special cases. We believe this approach offers a striking exposition of a difficult physical concept and can be used as a springboard for the development of more advanced concepts.

A second question raised is the more fundamental one of why the electric field is zero in the interior of the surface. All elementary textbooks, without exception, use the model of a conductor that contains an unlimited reservoir of positive and negative charge (so that any nonzero field will cause currents to flow). However, the argument is then extended to cover the case in which the conductor is hollow, and cavities obviously do not provide any such reservoir.

The computational approach outlined here predicts field cancellation on the interior of the modeled surface, assuming only that charge carriers can move freely through it. The assumption that the material itself can contribute dissociable charges appears not to be necessary; Coulomb's Law is sufficient. It is instructive and straightforward to replace the force between charges by a non-Coulombic law. When this is done, if this force falls off more quickly or more slowly than r^{-2}, the field in the interior does not vanish (clearly violating Gauss's law). In the former case, the charge density in the interior is not necessarily zero either.

Although the simulation demonstrates important outcomes of charge redistribution processes, it has some limitations. First, the number of charges represented is very much smaller than one would expect to find in any real situation, and thus the interior field cancellation is never exact. However, the result is certainly clear enough for students to infer the principle. They can readily observe that the field reduction becomes much more marked as they increase the number of charges. Second, the model can account for spontaneous charge redistribution but not for the induction of charge of opposite sign in the presence of external fields. The model could certainly be extended to include dissociable charges, but only at considerable computational load. Even so, we have a surprisingly sophisticated demonstration of electrostatic behavior.

Lastly, we believe that this work bears on another important issue. There has been much discussion since the invention of computers about the role of computation in physics—that is, is a computer simulation an experiment or a piece of theory? Haile[9] eloquently argues that "the role of simulation is not so much to predict (observation) but to make explicit how input and output variables are connected."

The current computation might be considered a thought experiment, in which the input assumption (Coulombic repulsion of like charges) is demonstrably connected to the output conclusion (the field is zero in the interior and strongest at sharp points on the surface). We believe that this example, and others like it, might contribute to the clarification of the role of computer simulations in physics.

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